Topic

3

Using Mathematical Models to Investigate Planetary Habitability

Activity A Finding a Mathematical Description of a Physical Relationship

Activity B Making a Simple Mathematical Model

Activity C The Role of Actual Data in Mathematical Models

Topic 3

Using Mathematical Models to Investigate Planetary Habitability

Overview

Students use mathematical computer models to describe the physical models constructed in Topic 2. These models will be derived from laboratory experiments (Activity A), from real world data (Activity B), and to varying degrees of complexity (Activity C). As powerful tools for science research, these models can help determine the climate system's dependence on variables that cannot easily be simulated in a laboratory.

Activity A: A simple mathematical model is derived by fitting a curve to observed data. Students will determine the mathematical relationship describing:
i) how the intensity of energy changes as a function of distance from a light source, and
ii) how the temperature changes as a function of distance from a light source.

Activity B: A simple mathematical model of the solar system is developed. Relationships derived in Activity A are used to determine the intensity of energy from the sun reaching the planets, and the temperatures of different planets.

Activity C: Adding complexity to models. Students will notice problems with results obtained in Activity B. Planetary albedo is introduced. A more complex model called *Global Equilibrium Energy Balance Interactive Tinker Toy* or GEEBITT will be employed to study the effect of adding this variable. An average albedo of earth and other planets will be determined with NASA data.

At the end of these activities, the students discuss what factors may influence a planet's temperature, other than distance and albedo. This leads to Topic 4, where the effect of a planet's atmosphere and the Greenhouse factor are considered.

Science Content

The relationship between data and mathematical models
Most scientists studying global warming use mathematical models to simulate the earth's atmosphere, oceans, and land surface processes. These mathematical models have advantages and disadvantages.

Activity A: Students use a physical model of a sun–planet system to obtain data describing relationships between (1) distance from the sun and intensity of light reaching the planet, and (2) between the planet's surface temperature and sun distance. Can observed data be described using a simplified mathematical relationship? How well can we describe physical interactions or conditions with mathematical relationships?

Activity B: To simulate the planet's temperatures we need to use a computer. Students use a simple spreadsheet program to develop their own computer model using the observed data collected from their physical model to set the initial conditions. While mathematical models take us to places where we cannot physically go, what are disadvantages of such models?

Earth's surface features and atmospheric properties influence planetary albedo
NASA satellite data is used to determine the average albedo of the earth. A simplified version of the GEEBITT is introduced to study factors that influence planetary albedo on Earth, Venus, Mars and Mercury. Characteristics of the actual and modeled planets are compared to assess the effectiveness of the model. This leads to a discussion of why albedo differs among the planets and which factors are most important in determining a planet's temperature and habitability.

Activity C: How does the earth's color affect its temperature? What about other planets? Is reflectivity or albedo important in determining a planet's temperature? How sensitive is the earth's temperature to changes in albedo?

Science Skills

Measurement and experimental design
Students design two experiments with physical models to determine the mathematical relationships between (1) distance from an energy source and intensity of energy at that distance, and (2) distance from an energy source and temperature of the object. Experiments are performed using solar cells and volt meters to collect data and determine these relationships.

Data analysis and trends in data
By graphing results of the two physical model experiments, students are able to relate the variables under consideration. A spreadsheet is used to replot the data, using some of the built-in analysis utilities, e.g., best-fit line.

Mathematical models, comparison of actual and model data, model development
The mathematical relationships discovered by the students are used to produce simple mathematical models in a spreadsheet program. Results are compared to actual data as part of the model development process. A second comparison with actual data reveals this as an iterative process. The final model is used to evaluate the relative effectiveness of various factors in maintaining planetary temperature and habitability.

Considering a Real World Problem

Deforestation and Urban Heat Islands

Earth's surface features are continually changing. In our planet's ancient past, natural events caused the earth's global mean temperature to cool to such an extreme that ice and snow covered much of its surface, producing an Ice Age. But today there is another variable to consider as a forcing on the planetary landscape and habitability – human civilization.

In Topic 3, students are introduced to the concept of planetary albedo. They determine mathematical relationships between several of the factors they have been studying that influence a planet's average temperature. They will be able to apply these understandings to consider two ways humans are altering Earth's surface features: 1) clearing land for agriculture and producing deforestation and 2) building cities, suburbs, and road systems that contribute to the Urban Heat Island Effect.

Students will be asked to consider how these two human impacts on Earth's land surface changes result in changes to surface and atmospheric albedo, as well as influence planetary temperature. By asking them to quantify and justify their responses based on their modeling activities in Topic 3, they can explain the scientific understandings they developed and apply them to current environmental problems.

Activity A

Finding a Mathematical Description of a Physical Relationship

4 class periods

In this first activity, students will derive a mathematical description (an equation) of the relationship between distance and the amount of energy received. Using this equation, they will produce a simple mathematical model that can determine the incident energy and the resulting surface temperature of a planet in the solar system. Emphasis should be placed on the process of taking observations, finding relationships, and modeling those relationships to describe situations that cannot be precisely duplicated in the laboratory.

An experiment design, with solar cells connected to volt meters, provides the data for students to measure the intensity of the light at certain distances perpendicular to a light source. Students record observations and plot them by hand on a graph. The relationships found in their graphs are described both in their own words and mathematically.

How do we obtain an "exact" mathematical description? A spreadsheet program is introduced to reproduce the hand plotted graph using a computer. Have students discuss the advantages of using a computer spreadsheet. The power of the spreadsheet is revealed when the students find the best-fit (linear and non-linear) for this data. The Inverse Square Law can be derived by generating the equation of the curve that best fits the data using the spreadsheet tools. By predicting the intensity at a point not yet tested, and then comparing that value to an actual measurement, students verify the validity of the equation.

Learning Objectives

By the end of this activity, students should be able to:

- ✓ Make measurements with a volt meter.
- ✓ Create a graph using a spreadsheet.
- ✓ Determine the best fit line for data using a spreadsheet.
- ✓ Quantitatively relate distance from a source to energy incident on an object.
- ✓ Quantitatively relate temperature of an object to the distance from a energy source.
- ✓ Use the derived relationships to predict observed values.

Materials

Light sources (150 watt light bulbs)
Support for light source
Solar cell/sunphotometers (Solar cells can be purchased from Radio Shack)
Digital thermometers, 1 volt meter
3 small plastic cups (to hold identical models)
Materials to make identical surfaces for each model
Computers loaded with Microsoft Excel software for the second part of this activity

Engagement

Class Period 1

Review the results from the hot and cold planet experiment conducted in Topic 2 by summarizing the observed relationships. Students may respond by saying, "the brighter the surface, the lower the temperature," or "the farther away the planet, the lower the temperature." Discuss how descriptions can be made more exact and detailed. To facilitate the discussion:

"Well, you say that if the distance gets larger, the temperature gets lower. Can you tell me how much lower the temperature will get if you double the distance? What would you need to know to answer this question?"

The need for more data should become clear, specifically the distance and the temperature or brightness at various distances to derive a more exact relationship.

Procedure

An experiment design challenge is presented to study the relationships between distance, temperature, and intensity of light at different distances. Distribute handouts for students to complete: (1) Team Members, (2) Experimental Design Proposal, and (3) Methodology for a Controlled Experiment. Review the function of a solar cell and volt meter as materials to be used in the experiment. Divide the class into teams of five. Each team must have a leader and two sub-teams, one to measure the distance–temperature relationship, another to measure the distance–intensity relationship.

Class Period 2

Distribute the Data Sheets: Physical Model Experiments, for students to record the data collected from the physical model. The instructor can circulate around the room and provide help or suggestions as needed. Students work in their laboratory groups to complete the handout Experimental Results: Physical Model Experiments. Part of this handout involves the use of a hand drawn graph to determine mathematical relationships from the experimental data.

Consensus

Class Period 3

Following some discussion of the physical model results, introduce the spreadsheet computer program. Distribute the handout Experimental Results: Deriving Quantitative Relationships with a Spreadsheet. This serves as a guide for plotting results with a spreadsheet program. Students compare the results graphed by hand and the spreadsheet. Collectively, all the investigation groups decide the best way to represent the relationships from their experiments.

Synthesis

Class Period 4

The activity concludes with a discussion of results produced in the two experiments. What is the relationship between distance and temperature or distance and intensity that the students could not say before the experiment and the plotting exercise? How reliable are the results? The Investigation Questions should be completed.

Activity B

Making a Simple Mathematical Model

3 class periods

Using their observed results for the relationship between distance and the energy reaching an object, students will make a simple mathematical model with a computer spreadsheet program. These models will be developed with the capability to change the luminosity and distance of their light source and to calculate the energy reaching the object, and its black body temperature. Experiments will be designed to obtain the theoretical black body temperatures of Earth, Venus and Mars. By comparing model results to the actual temperature values, students can continue to discuss planetary differences, the role of models in science and possible improvements to their spreadsheet model.

Learning Objectives

By the end of this activity, students should be able to:

- ✓ Make a spreadsheet model using Excel to determine the temperature of an object given its distance from an energy source.
- ✓ Compare theoretical and observed black body temperatures for a planet.
- ✓ State three advantages of using mathematical models.
- ✓ Describe three disadvantages of using mathematical models.

Materials

Computer loaded with Microsoft Excel software, and the equations derived by students in Activity A

Engagement

Class Period 1

Students review the results from the Radiation Balance Model computer simulation. All the scenarios achieved equilibrium temperature, but the farther away the object, the lower the equilibrium temperature as less energy reaches the object. Students should be able to describe the Inverse Square Law qualitatively. Ask: How did the simulation achieve these results? How would they go about making a simulation that could give results similar to that of the Radiation Balance Model simulation?

Procedure

Review the concept that mathematical models use equations to determine their results. These equations describe relationships observed in nature, such as the Inverse Square Law. The more complicated the relationship to be simulated, the more sophisticated the model must be. Students go through the introductory activity of how to use a spreadsheet.

Class Period 2

Students use the mathematical equations describing the effect of distance from a source upon (1) the intensity of light reaching an object at that distance, and (2) the temperature of the object. They will build a simple mathematical model that determines the black body surface temperature of a planet given its distance from the sun. The luminosity of the sun and the solar constant at the surface of the sun are provided. Students are then assigned the task of building their spreadsheet model while referring to the instructions provided in the handout: Learning Excel: Applying Features of a Spreadsheet Program to Create a Simple Planetary Model.

Class Period 3

Students complete the spreadsheet models and determine the black body temperatures of Earth, Venus and Mars. Results are compared to the observed values and the role of models in science investigations is discussed. At this point, the instructor may introduce the Inverse Square Law and the Stefan-Boltzmann Law as the "actual" relationships and have students critique their own equations.

Consensus

A comparison of the spreadsheet models is conducted. Students discuss how predictions made for the solar constants and surface temperatures for Venus, Earth, Mars and Mercury compare to the actual values. By determining the percent error of their predictions, they can discuss which models did a better job predicting the true values, and why. What made some models better than others? What were the strengths and weaknesses? What are the advantages and disadvantages of using spreadsheets to model planets?

Synthesis

After completing the Investigation Questions, the class reconvenes to discuss their responses. In discussing model improvements, it should be emphasized that a model is never finished. Its continued development is an effort to improve our understanding of the processes being simulated.

Extra Credit: Provide students with the Inverse Square Law and the Stefan-Boltzmann Law and ask them to include these relationships in their spreadsheet models. They should modify their spreadsheets and then compare the new model to older versions.

Activity C

The Role of Actual Data in Mathematical Models

1 class period

One of the improvements that students should suggest making to their spreadsheet models is to include the albedo of the planet. Students will use an advanced spreadsheet model called *Global Equilibrium Energy Balance Interactive Tinker Toy* or GEEBITT, that includes an albedo factor as well as some other features, to investigate the effect of the planet's albedo. In order to properly utilize this model, students will be first asked to determine the earth's albedo by examining NASA data. They will then use a spreadsheet called *Albedo Calculator* to determine the value of the albedo to use in GEEBITT. The resulting model values for the gray body surface temperatures of Earth, Venus and Mars are again compared to the actual values of these temperatures. (*Note: A "Black Body" is a perfect emitter and absorber of energy. A "White Body" is an object that reflects all of the energy it receives and therefore absorbs none of that energy. Both Black and White bodies are ideal objects and do not actually exist, though some materials come close to these behaviors. Objects in the universe are really "Gray Bodies", both reflecting and absorbing some of the energy they receive.*) The students are then asked to propose explanations for observed discrepancies and suggestions on how to improve the model.

Learning Objectives

By the end of this activity, students should be able to:

- ✓ Read and analyze an albedo map of the earth.
- ✓ Determine the average albedo of the earth.
- ✓ Become familiar with and utilize the GEEBITT spreadsheet model.
- ✓ Compare theoretical and observed gray body temperatures for a planet.
- ✓ Suggest further improvements to their spreadsheet models.

Materials

Computers loaded with Microsoft Excel software
Plots of the earth's albedo as seen by satellites
Spreadsheets Albedo Calculator and GEEBITT. These can be downloaded from the web site: http://icp.giss.nasa.gov/education

A user's guide to GEEBITT can be found in the Appendix of the Student Activities.

Engagement

The instructor should direct students to their suggestions from the previous class period on possible improvements of their spreadsheet models. One of the suggestions will probably have been the inclusion of albedo. To do this, we need to modify the model and determine a value to use for the albedo of the planet. At this point the instructor can review the concept of albedo and show the class an albedo map of the earth obtained from satellite observations and have the students try to interpret the map. Students will see that there are a variety of albedos, and can be asked which value could be used to represent the albedo of the entire planet. Someone may suggest using the average value, and the teacher can ask how that would be done. After students suggest a possible procedure, the teacher can explain the albedo calculator map activity.

Procedure

Students are divided into groups of five. Each group is assigned a portion of the earth and instructed to determine the percentage of their portion that is ocean, vegetation, desert, or ice. The percentages are entered into the Albedo Calculator spreadsheet to determine the average albedo of that portion. The average albedos of each portion are then collectively averaged in order to obtain the albedo of the earth.

After the students have determined the average albedo, the teacher can ask if they are done, or if there is something else that needs to be included. If they are unable to answer, the teacher can show them photographs of Earth taken from space. At this point some students should point out that we have not included the clouds and the effect they have on albedo. The students are then given a plot of the average cloud cover in their region, and asked to determine the effect of the clouds on albedo and incorporate this into the Albedo Calculator. A value of the average albedo of the earth including cloud effects is finally derived.

Students then use this value as well as the albedos of Mars and Venus in mini-GEEBITT, Version A, which includes albedo. Students obtain their theoretical gray body temperatures of these three planets.

What are the gray body surface temperatures of Venus, Earth, and Mars?

Students compare their model results with the actual surface temperature of the three planets. The discrepancies are now found to be even greater (particularly with Venus' cloud cover), and the students are asked to explain the inadequacies of this model.

Consensus

Have students suggest other modifications that may be necessary to improve the model's simulation of real planets.

Synthesis

Once again, students are asked why the temperatures of the simulated planets still do not agree with observed values. A concluding discussion follows and they should complete the Investigation Questions as homework.

Evaluating Student Work

Topic 3. Using Mathematical Models to Investigate Planetary Habitability

Activity A – Investigation Questions

1. Did the changing distance affect the temperature and the intensity of the light as you predicted? Explain.

2. What advantages did the spreadsheet offer over analyzing your data by hand?

3. Do your final equations accurately describe the relationships between distance and temperature, and distance and intensity? Why or why not?

4. Now that you have mathematical relationships between distance, intensity, and temperature, how can you use them to study planetary conditions in the solar system?.

5. Engineers are given the task to light Yankee Stadium for night games. How can they go about deciding the intensity and distance of the lights from the field?

6. An architect wants to know if the design for a 10 story building for downtown Los Angeles will be able to withstand a strong earthquake. What options are there for testing the design? What are the advantages and disadvantages of the options?

Answer Guide – Questions 1 through 6

Students are expected to comment on the usefulness of physical and mathematical models and the uncertainties of experimental data. It is important to understand that once derived, mathematical relations have the advantage that they can be applied to all kinds of different situations.

Activity B – Investigation Questions

1. Does your model produce temperature differences between the planets that agree with your expectations? Explain.

2. Do you think that modeling planets as black bodies provides an accurate representation of the planets' temperatures? If not, how can you improve the model to derive more accurate results?

3. All weather predictions are done with mathematical models, and some predictors do better jobs the others. Give some possible reasons why the Acme Weather Company (not the company's real name) has the worst record for predicting the weather.

Answer Guide – Questions 1 through 3

The students are expected to understand that the black body temperature is only an assumption to illustrate the effect of the distance from the light source on the temperature of the planet, and is the first part in a process to calculate the correct temperature. They should also understand that mathematical models are only as good as the input that goes into the calculation and the accuracy of the equations that are used.

Activity C – Investigation Questions

1. How does the value for the average surface temperature of the earth without albedo compare with the value you obtained with your own spreadsheet model? Is this what you would expect?

2. Compare the value of the surface temperature of the earth without albedo with the average surface temperature with albedo. Is this what you would expect? Explain your reasoning.

3. Examine and compare the References: World Albedo Maps for January and July. Answer the questions below:

   a) What similarities do you see between the two maps? Are these expected? Explain.

   b) What are the major differences between the two maps? Give an explanation as to why you think these differences occur.

   c) In the past, the earth has been immersed in a series of ice ages. What changes would you expect to see in an albedo map of the earth during one of these ice ages?

4. What effect, if any, do you think the presence of humanity has upon the albedo of the earth? Explain your answer.

5. The actual average temperature at the earth's surface is 15ºC, that of Venus is 430ºC, and that of Mars is – 45ºC. How do the temperatures with albedo compare to the actual surface temperatures? What other factors may need to be modeled in GEEBITT in order to calculate a better surface temperature estimate?

6. Do you think it is possible to produce a model that is an exact simulation of the real world? Defend your answer.

Answer Guide

Questions 1 through 5

The students should understand that higher albedo means cooler temperature and that elements like ice and snow on the earth's surface increase the planetary albedo. Also human intervention with acts like deforestation can have an effect on the planet's albedo.

Questions 6 and 7

Students are expected to understand that including the albedo in the calculation of the surface temperature of a planet adds one more degree of complication to the calculation but does not get us any closer to the right answer. Based on their experience with the physical models they should hypothesize that the missing ingredient that will give the right surface temperatures has something to do with the presence of an atmosphere around the planets.

Essay: Real World Problem – Deforestation and Urban Heat Islands

The earth's surface features are continually changing. In our planet's ancient past, natural events caused the earth's global mean temperature to cool to such an extreme that ice and snow covered much of the planet's surface, producing an Ice Age.

But today, there is another variable to consider as a forcing on the planetary landscape and habitability: human civilization.

Land is cleared for agricultural production and to raise cattle. Loggers cut down trees for commercial profit from products manufactured with wood. This process of deforestation is decreasing the world's forests. Tropical forests are particularly impacted. Some reports estimate that these areas will disappear in 100 years if the current rate of deforestation continues (NASA, 1998). The Brazilian Congress is voting to clear 50% the Amazon forest for agriculture and cattle. They believe this plan will improve the country's poor economy and enhance personal economic prosperity of its citizens. What effects might deforestation have on Earth's global and local temperatures?

Another way humans are changing Earth's surface is by building structures on the landscape. In the 20th century, great cities have developed. Areas once covered with plants and trees are now sites of skyscrapers, buildings, and roads. These large cities are associated with a phenomenon called *Urban Heat Islands*, where temperatures warm due to built-up environments. Why would temperatures increase as a city grows?

Assignment: Write a 300 word scientific essay that addresses temperature changes as a result of variations in albedo on the Earth's surface and atmosphere. How will replacement of forests with agriculture or pastures affect albedo? Do you think large-scale deforestation in the Amazon will significantly influence local and global temperatures? Would this cause warming or cooling? What are other important factors besides albedo that control temperature? On a smaller scale, cities replace vegetation with buildings, producing Urban Heat Islands. Why does the city warm as reflectivity increases? Justify your responses using evidence from your experiments with the Albedo Calculator and GEEBITT. Include examples found on the Internet of areas where humans may be changing albedo and temperature. Refer to the References: Regional Effects of Human Development – Deforestation, and Regional Effects of Human Development – Urban Heat Islands.

Minimum Expectations for this Essay

- ✓ Students should explain what they think the effect of removing forests will have on the albedo of those areas.
- ✓ Students should explain what they expect large-scale deforestation of the Amazon will have on local and global temperatures.
- ✓ Students should describe at least two other factors that would affect temperatures locally and globally.
- ✓ Students should give at least three reasons why temperatures of cities increase as the cities increase in size.
- ✓ Students should justify these reasons with evidence from their experiments and other named sources.